

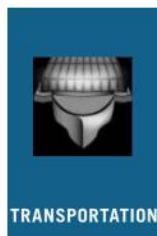
Algeria applies dynamic performance estimate to pipeline system rehab

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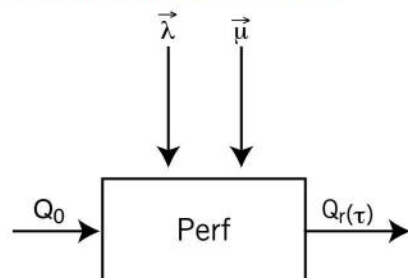
New methods of dynamically estimating complex natural gas systems performance can prove useful not only in design and operations, but also in planning for rehabilitation and expansion.

Algeria has a complex network of natural gas pipelines moving production to national and international customers. Its pipeline operators have proposed an ambitious program of rehabilitation and expansion. The very high costs of such programs, due mainly to network complexity, required development of tools to locate bottlenecks in advance and eliminate them as efficiently as possible.

After selecting a framework defining objectives, planners turned to developing a model that would allow them to quantify the impact of any decision on meeting these objectives. This article applies the concept of performability¹⁻³ to this process.

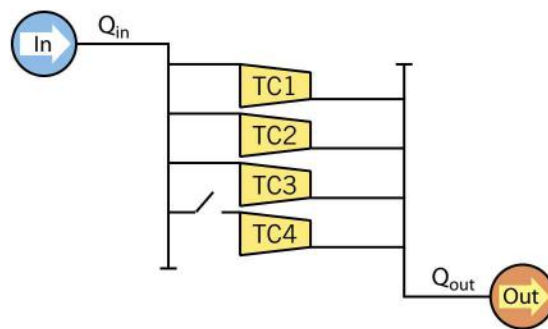
PERFORMABILITY BLACK BOX

FIG. 1



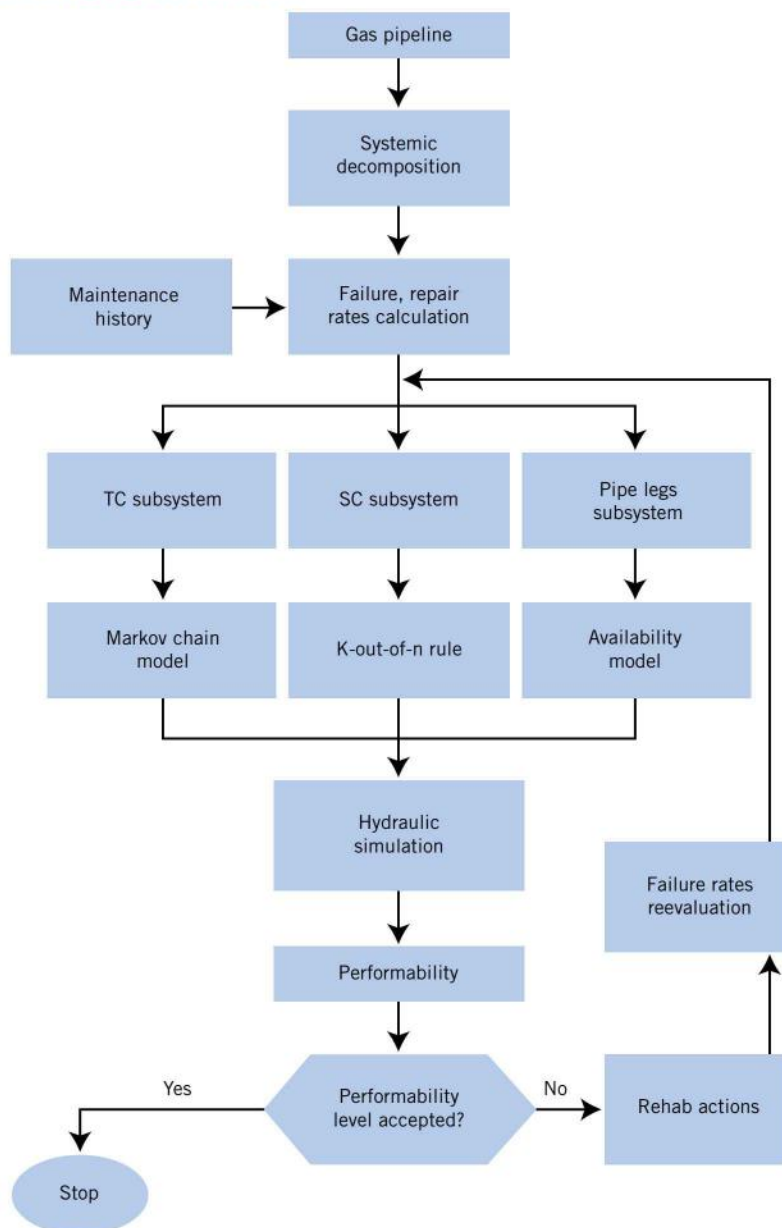
3+1 TC COMPRESSOR STATION

FIG. 2



METHODOLOGY FRAMEWORK

FIG. 3



For complex pipeline networks with numerous redundancies, the failure of any single element does not cause a total stop in operations but a decline in performance characterized by reduced flow. Work continues, even in the presence of failures. This kind of system is degradable and fault-tolerant. Its correct modeling must integrate reliability and performance in the same reference frame.

Degradable systems' operating assumptions include the ability to remain in service at various levels of performance. At system start-up, all components are assumed to be operational and the system will operate at maximum performance. When a component fails, the system will reconfigure itself and restart activities, albeit with degraded performance. The long interval typical between failures means the system will operate mostly in steady state between successive reconfigurations and failures.

Background

Performability is the association of a reliability model with a performance model. The first task is defining the mathematical nature of this association.

In a black box representation (Fig. 1) with the rated capacity of the system, Q_0 , an input variable and an output variable of real capacity, Q_r , the system process is subject to two disturbance vectors: failure rates, $\vec{\lambda}$, and repair rates $\vec{\mu}$. Rated capacity is the transport capacity of the network in absence of failures. Failure's random nature leads to random fluctuations in flow rate over time, τ (Equation 1). Statistical distribution can then model these fluctuations, or they can be assessed as random processes with discrete space-of-failure states (Equation 2).

Natural gas pipelines generally encompass compressor stations (CS) and pipe legs. A compressor station includes several turbo-compressors (TC) installed, in most cases, on passive redundancy. Algerian gas compressor stations are generally configured with 3+1 TC installed in parallel on passive redundancy (Fig. 2).

EQUATIONS

$$\text{Perf} = \frac{1}{Q_0} \left[\frac{1}{\tau} \int Q_r(\tau) d\tau \right] = \frac{1}{Q_0} M\{Q_r\} \quad (1)$$

$$\text{Perf} = \frac{1}{Q_0} \sum_{k=0}^S P_k Q_k \quad (2)$$

Where:

Q_0 = nominal flow rate

$M(Q_r)$ = expected value of variable Q_r

P_k = probability of failure state

Q_k = flow rate associated with failure state

S = total number of failure states

$$\lambda = \left[\frac{1}{k} \sum_{i=1}^k \text{UT}_i \right]^{-1} \quad (3)$$

$$\mu = \left[\frac{1}{k} \sum_{i=1}^k \text{TTR}_i \right]^{-1} \quad (4)$$

Where:

TTR_i = repair duration after failure i

k = total number of failures during the observation period

$$\frac{dP_1}{dt}(t) = -3\lambda P_1(t) + \mu P_2(t) \quad (5)$$

$$\frac{dP_2}{dt}(t) = -(\mu + 3\lambda) P_2(t) + 3\lambda P_1(t) + 2\mu P_3(t) \quad (6)$$

$$\frac{dP_3}{dt}(t) = -(2\lambda + 2\mu) P_3(t) + 3\lambda P_2(t) + 3\mu P_4(t) \quad (7)$$

$$\frac{dP_4}{dt}(t) = -(\lambda + 3\mu) P_4(t) + 2\lambda P_3(t) + 4\mu P_5(t) \quad (8)$$

$$\frac{dP_5}{dt}(t) = 4\mu P_5(t) + \lambda P_4(t) \quad (9)$$

Where:

$P_1(0) = 1$ and $P_2(0) = P_3(0) = P_4(0) = P_5(0) = 0$

$$P_m = P(E_1) + P(E_2) \quad (10)$$

$$P_{m1} = P(E_3), P_{m2} = P(E_4) \quad (11)$$

$$q_k = \frac{C^k \cdot \lambda^k \cdot \mu^{n-k}}{(\lambda_{cs} + \mu_{cs})^n} \quad (12)$$

$$\text{POS}_i = \frac{\mu_i}{\mu_i + \lambda_i L_i} \quad (13)$$

Where:

POS_i = pipe leg operating state probability

λ_i = pipe leg failure rate in $\text{h}^{-1}\text{Km}^{-1}$

μ_i = pipe leg repair rate in h^{-1}

L_i = pipe leg length in km

GAS PIPELINE SYSTEM, HIERARCHICAL COMPOSITION

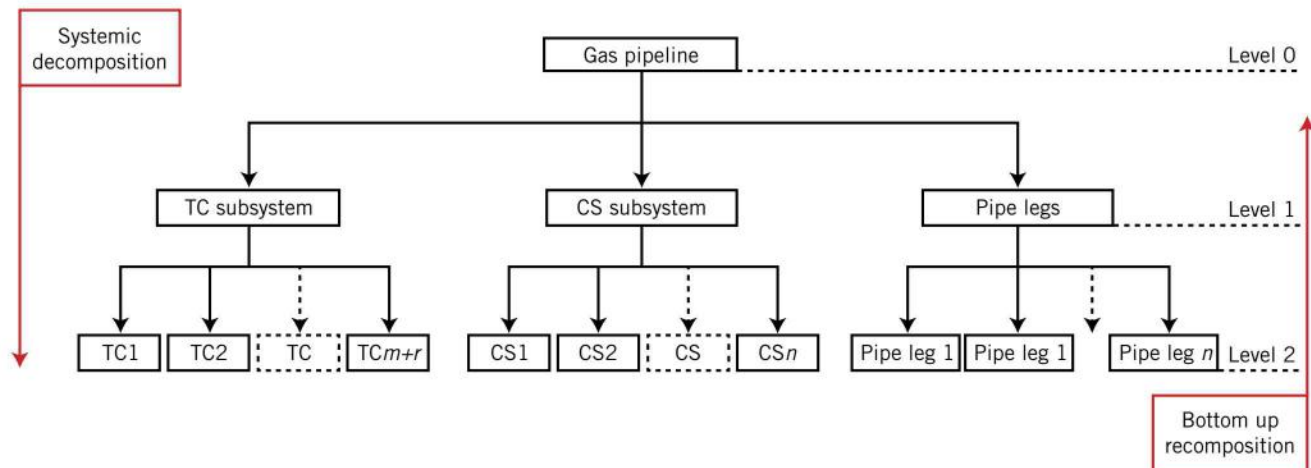


FIG. 4

MARKOV DIAGRAM, SYSTEM STATE PROBABILITIES

FIG. 5

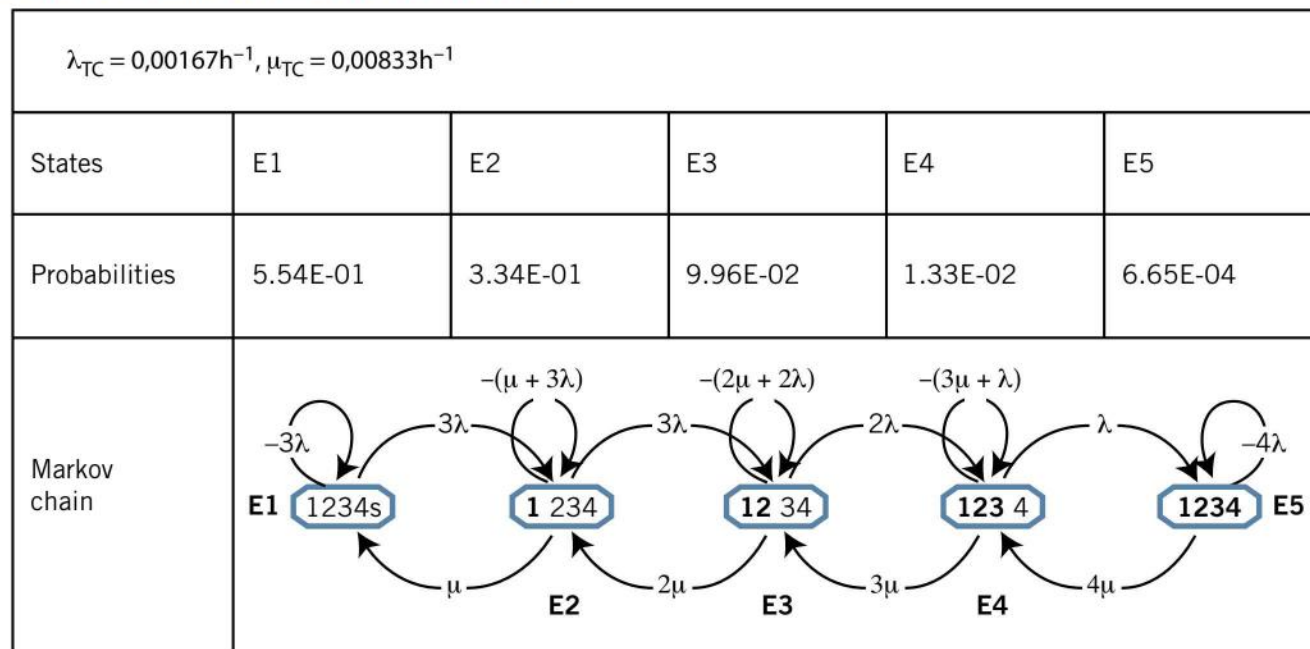


Fig. 3 shows the general approach used for this article, taking into account the following assumptions:

- Failure and repair rates are the same for all TC.
- Failure and repair rates are the same for all CS.
- Failure and repair rates, issued from databases, are constant for all pipe legs.
- Repairs start immediately after failures.

Systemic approach

For complex pipeline systems, the main drawbacks regarding Equation 2 are:

- The huge dimension of space, S .
- Difficulties related to defining probabilities for every state.

Complex pipeline systems therefore require a systemic approach to bring space, S , to manageable dimensions⁴⁻⁶ by splitting the global system into subsystems. Every subsystem

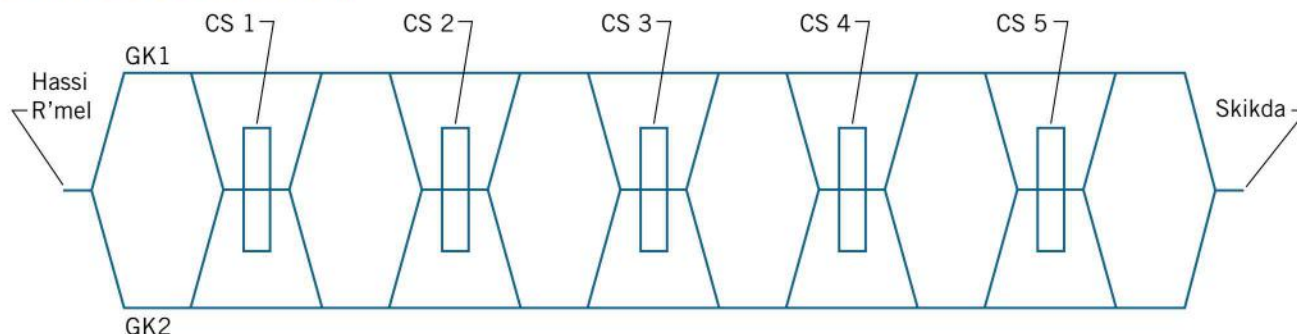
can similarly split into lower level subsystems. A subsystem generally will be defined as an interacting collection of elements or lower level subsystems.

An element forms the basis of a subsystem that can't be broken down into further sub-elements. A hierarchical flow diagram having more or fewer high-level numbers depending on the degree of detail required from the modeling process illustrates this breakdown. This article uses a three-level hierarchy to model pipeline reliability (Fig. 4).

A bottom-up approach reconstitutes the global model. An element can be either operational or in a failure state. An element in good operating condition can be in operation or on standby. A combination of a subsystem's constituent elements might characterize its state. In the same manner, combining immediately lower subsystem states can define a high-level subsystem state.

GK1-GK2 GAS PIPELINE SYSTEM

FIG. 6



Reliability rates

The basic reliability indexes of compression unit components are failure rates, TC, CS λ_{TC} , and λ_{CS} , and repair rates, μ_{TC} and μ_{CS} . The law of large numbers (LLN) provides the basis for the “frequentist approach”⁶⁻¹⁰ representing the classical identification of reliability indexes.

Equation 3 estimates the rate of failure for UT_1, UT_2, \dots, UT_k as a sample of the random variable representing the up times of selected equipment. The same logic applies to the repair rate (Equation 4).

Failure modeling

A compressor station consists of n operating and r standby TC. Each TC is in either a good operating state (OS) or failure state (FS). Different combinations of these elements determine finite set E , with C_{n+r}^2 dimension, for all possible system states. A stochastic modeling process using Markov chain¹¹⁻¹² can evaluate probabilities of different E states. Circles on the Markov model represent the component states (working or failed), with arrows showing the direction of transition between states (failure or repair). Arrows with numeric values show failure or repair rates.

System states are a combination of different element states. A change in the condition of any element starts system transition from one state to another. The homogeneous Markovian process features constant transition rates with the following assumptions:

- Failure rates identical for $m+r$ TC.
- Perfect standby element permutation.
- Immediate spare part availability.
- $m+r$ repair teams.

Fig. 5 shows the 3+1 TC. Permutations include:

- E1 : 3+1 TC in OS.
- E2 : 3 TC in OS, 1 TC in FS.
- E3 : 2 TC in OS, 2 TC in FS.
- E4 : 1 TC in OS, 3 TC in FS.
- E5 : 3+1 TC in FS

CS SUBSYSTEM STATE PROBABILITIES

Table 1

$$\lambda_{CS} = 0.0025 \text{ hr}^{-1}, \mu_{CS} = 0.0417 \text{ hr}^{-1}$$

CS state	Probability	CS state	Probability
0 CS in FS, q_0	0.747258173	All CS in OS	0.747258173
1 CS in FS, q_1	0.224177452	CS i in FS q_{1i}	0.04483549
2 CS in FS, q_2	0.026901294	CS i , CS j in FS q_{2ij}	0.002690129
3 CS in FS, q_3	0.001614078	CS i , CS j , CS k in FS q_{3ijk}	0.000161408
4CS in FS, q_4	4.84223E-05	CS i , CS j , CS k , CS l in FS	9.68447E-06
5 CS in FS, q_5	5.81068E-07	All CS in FS	5.81068E-07

EVENT PROBABILITY MODELS

Table 2

EVENTS	PROBABILITY MODELS
E_0 : all pipe legs in OS + all the CS in OS + at least m TC of $m+r$ in OS for all CS	$P(E_0) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_0 P_m^5$
E_{1i} : pipe leg i in FS + all the CS in OS + at least m TC of $m+r$ in OS for all CS	$P(E_{1i}) = \text{PF}_i \frac{\prod_{s=1}^{N_p} \text{POS}_s}{\text{POS}_i} q_0 P_m^5 \quad i = 1 \div N_p$
E_{2a} : all pipe legs in OS + all the CS in OS + $m-1$ TC in OS in CS i + at least m TC of $m+r$ in OS for all other CS	$P(E_{2a}) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_0 (P_{m-1})_i P_m^4 \quad i = 1 \div N_{CS}$
E_{3i} : all pipe legs in OS + CS i in FS + at least m TC of $m+r$ in OS for all CS	$P(E_{3i}) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_{1i} P_m^5 \quad i = 1 \div N_{CS}$
E_{4ij} : all pipe legs in OS + 2 CS i and j in FS + at least m TC of $m+r$ in OS for all CS	$P(E_{4ij}) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_{2ij} P_m^5 \quad i = 1 \div N_{CS} - 1 \text{ and } j = i + 1 \div N_{CS}$
E_{5ij} : all pipe legs in OS + CS i in FS + $m-1$ TC in OS in CS j + at least m TC of $m+r$ in OS for all other CS	$P(E_{5ij}) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_{1i} (P_{m-1})_j P_m^4 \quad i = 1 \div N_{CS} \text{ et } j = 1 \div N_{CS}$
E_{6ij} : all pipe legs in OS + all CS in OS + $m-1$ TC in OS in two different CS i and j + at least m TC of $m+r$ in OS for all other CS	$P(E_{6ij}) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_0 (P_{m-1})_i^2 P_m^3 \quad i = 1 \div N_{CS} - 1 \text{ and } j = i + 1 \div N_{CS}$
E_{7i} : all pipe legs in OS + all CS in OS + $m-2$ TC in OS in CS i + at least m TC of $m+r$ in OS for all other CS	$P(E_{7i}) = \left(\prod_{s=1}^{N_p} \text{POS}_s \right) q_0 (P_{m-2})_i P_m^4 \quad i = 1 \div N_{CS}$
E_{8ij} : pipe leg i in FS + CS j in FS + at least m TC of $m+r$ in OS for all CS	$P(E_{8ij}) = \text{PF}_i \frac{\prod_{s=1}^{N_p} \text{POS}_s}{\text{POS}_i} q_{1j} P_m^5 \quad i = 1 \div N_p \text{ and } j = 1 \div N_{CS}$
E_{9ij} : pipe leg i in FS + all the CS in OS + $m-1$ TC in OS in CS j + at least m TC of $m+r$ in OS for all other CS	$P(E_{9ij}) = \text{PF}_i \frac{\prod_{s=1}^{N_p} \text{POS}_s}{\text{POS}_i} q_0 (P_{m-1})_j P_m^4 \quad i = 1 \div N_p \text{ and } j = 1 \div N_{CS}$
E_{10ijk} : pipe legs i and j in FS + all CS in OS + at least m TC of $m+r$ in OS for all CS	$P(E_{10ijk}) = \text{PF}_i \cdot \text{PF}_j \frac{\prod_{s=1}^{N_p} \text{POS}_s}{\text{POS}_i \text{POS}_j} q_0 P_m^5 \quad i = 1 \div N_p - 1 \text{ and } j = i + 1 \div N_p$

Where:

- P_m^5 = probability at least m TC of $m+r$ in operating state in CS
- P_{m-1}^5 = probability to have $m-1$ TC in operating state in CS
- P_{m-2}^5 = probability to have $m-2$ TC in operating state in CS
- q_{1i} = probability of one CS in failure state
- q_{2ij} = probability of all CS in operating state
- POS = pipe leg operating state probability
- PF = pipe leg failure state probability
- N_p = number of pipe legs
- N_{CS} = number of CS

BASE-CASE GK1-GK2 PERFORMABILITY COMPUTATION

Table3

Pipeline failure state	Probability	Q_k million cu m/hr	Pipeline failure state	Probability	Q_k million cu m/hr
All pipe legs in OS + all CS in OS + at least 3 TC in OS for all CS	4.10E-01	3.0	Pipe leg 6 GK2 in FS+ all CS in OS + at least 3 TC in OS for all CS	2.26E-04	1.53
All pipe legs in OS + CS 1 in FS + at least 3 TC in OS for all CS	2.46E-02	2.7	All pipe legs in OS + CS1 & CS2 in FS + at least 3 TC in OS for all CS	1.48E-03	2
All pipe legs in OS + CS 2 in FS + at least 3 TC in OS for all CS	2.46E-02	2.53	All pipe legs in OS + CS1 & CS3 in FS + at least 3 TC in OS for all CS	1.48E-03	2.5
All pipe legs in OS + CS 3 in FS + at least 3 TC in OS for all CS	2.46E-02	2.64	All pipe legs in OS + CS1 & CS4 in FS + at least 3 TC in OS for all CS	1.48E-03	2.5
All pipe legs in OS + CS 4 in FS + at least 3 TC in OS for all CS	2.46E-02	2.6	All pipe legs in OS + CS1 & CS5 in FS + at least 3 TC in OS for all CS	1.48E-03	2.4
All pipe legs in OS + CS 5 in FS + at least 3 TC in OS for all CS	2.46E-02	2.49	All pipe legs in OS + CS2 & CS3 in FS + at least 3 TC in OS for all CS	1.48E-03	2
All pipe legs in OS + all CS in OS + 2 TC in OS in CS 1	4.60E-02	2.95	All pipe legs in OS + CS2 & CS4 in FS + at least 3 TC in OS for all CS	1.48E-03	2.5
All pipe legs in OS + all CS in OS + 2 TC in OS in CS 2	4.60E-02	2.95	All pipe legs in OS + CS2 & CS5 in FS + at least 3 TC in OS for all CS	1.48E-03	2.4
All pipe legs in OS + all CS in OS + 2 TC in OS in CS 3	4.60E-02	2.95	All pipe legs in OS + CS3 & CS4 in FS + at least 3 TC in OS for all CS	1.48E-03	2.1
All pipe legs in OS + all CS in OS + 2 TC in OS in CS 4	4.60E-02	2.95	All pipe legs in OS + CS3 & CS5 in FS + at least 3 TC in OS for all CS	1.48E-03	2.4
All pipe legs in OS + all CS in OS + 2 TC in OS in CS 5	4.60E-02	2.95	All pipe legs in OS + CS4 & CS5 in FS + at least 3 TC in OS for all CS	1.48E-03	2
Pipe leg 1 GK1 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.31E-04	2.2	All pipe legs in OS + all CS in OS + 2 TC in OS in CS1 & CS2	5.16E-03	2.93
Pipe leg 2 GK1 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.51E-04	2.15	All pipe legs in OS + all CS in OS + 2 TC in OS in CS1 & CS3	5.16E-03	2.93
Pipe leg 3 GK1 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.44E-04	2.15	All pipe legs in OS + all CS in OS + 2 TC in OS in CS1 & CS4	5.16E-03	2.94
Pipe leg 4 GK1 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.32E-04	2.2	All pipe legs in OS + all CS in OS + 2 TC in OS in CS1 & CS5	5.16E-03	2.94
Pipe leg 5 GK1 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.20E-04	2.2	All pipe legs in OS + all CS in OS + 2 TC in OS in CS2 & CS3	5.16E-03	2.9
Pipe leg 6 GK1 in FS+ all CS in OS + at least 3 TC in OS for all CS	2.26E-04	1.74	All pipe legs in OS + all CS in OS + 2 TC in OS in CS2 & CS4	5.16E-03	2.93
Pipe leg 1 GK2 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.31E-04	1.84	All pipe legs in OS + all CS in OS + 2 TC in OS in CS2 & CS5	5.16E-03	2.94
Pipe leg 2 GK2 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.51E-04	1.77	All pipe legs in OS + all CS in OS + 2 TC in OS in CS3 & CS4	5.16E-03	2.88
Pipe leg 3 GK2 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.44E-04	1.78	All pipe legs in OS + all CS in OS + 2 TC in OS in CS3 & CS5	5.16E-03	2.9
Pipe leg 4 GK2 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.32E-04	1.8	All pipe legs in OS + all CS in OS + 2 TC in OS in CS4 & CS5	5.16E-03	2.86
Pipe leg 5 GK2 in FS+ all CS in OS + at least 3 TC in OS for all CS	1.20E-04	1.86	Gas pipeline performability without rehabilitation, 0.8934		

Equations 5-9 comprise the state equations system associated with this example.

Equation 10 shows the probability that at least 3 TC in the CS are operating.

Equation 11 shows the probability of having 2 or 1 TC operating.

Fig. 5 presents the solution obtained using the model described, with TC in failure state shown in bold.

Failure probability

Subsystem CS is considered separately from subsystem TC. A CS failure does not mean failure of all TC. Even if all TC of a CS are in good operating condition, however, the possibility of a compressor station breakdown remains. CS failures are usually due to secondary components, particularly control chains and electrical supply equipment. Assuming an identical reliability index λ_{CS} and μ_{CS} for all compressor

RELIABILITY INDEXES

Table 4

	Base case	Post-rehabilitation		
		TC	CS	Pipe leg
λ_{TC} (h^{-1})	1.67E-03	7.14E-04	1.67E-03	1.67E-03
μ_{TC} (h^{-1})	8.33E-03	8.33E-03	8.33E-03	8.33E-03
λ_{CS} (h^{-1})	2.50E-03	2.50E-03	1.25E-03	2.50E-03
μ_{CS} (h^{-1})	4.17E-02	4.17E-02	4.17E-02	4.17E-02
λ_l ($h^{-1} \text{ Km}^{-1}$)	8.00E-08	8.00E-08	8.00E-08	1.14E-08
μ_l (h^{-1})	2.08E-02	2.08E-02	2.08E-02	2.08E-02

GK1-GK2 POST-REHABILITATION PERFORMABILITY

Table 5

Rehabilitation strategy	Performability	Expected throughput, billion cu m/year
Without rehab	0.8934	22.514
Rehab of TC subsystems	0.9526	24.007
Rehab of CS subsystems	0.9216	23.224
Rehab of pipe legs subsystem	0.8950	22.554
Rehab of TC subsystems + CS subsystems	0.9735	24.534
Rehab of TC subsystems + pipe legs subsystem	0.9540	24.043
Rehab of CS subsystems + pipe legs subsystem	0.9231	23.263
Rehab of TC subsystems + CS subsystems + pipe legs subsystem	0.9749	24.569

stations, using the k-out-of-n rule, Equation 12 shows the probability to get k CS out of service.^{4,6}

Table 1 presents an application of this model for the natural gas transmission pipeline linking Hassi R'mel to Skikda with 5 CS.

Equation 13 determines the probability that a pipe leg is in a good operating state (OS).

Reliability model

Gas pipelines of low or average length, such as the Algerian pipelines, are unlikely to suffer a simultaneous failure of more than two elements. Elementary probabilities that use fundamental theorems of probabilities theory calculate the probability of each event. Table 2 shows statistically significant events and their corresponding probability models. . .

Hydraulic simulation

Each failure state, E_k , has an associated flow rate, Q_k . Hydraulic relations describing the steady-state gas flow with-

in a pipeline combined with performance characteristics of the TC can determine flow rate. Fig. 6 shows the results of applying SIMONE gas pipeline simulation software to this task.

The last column of Table 3 shows the results from applying SIMONE to a gas pipeline supplying the Skikda LNG plant from Hassi R'mel field. Table 3 also shows the most statistically significant results derived, during consideration of 1-2 simultaneous failures on gas pipeline GK1/GK2, out of 259 different combinations. The data reported in Table 4 yield the results obtained for various rehabilitation strategies shown in Table 5.

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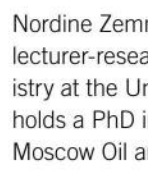
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